

# Large $N_c$ Behavior of Light Scalar Meson Nonet Revisited

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## Abstract

We study whether light scalar meson nonet can survive in large values of  $N_c$  through observing how scattering amplitudes behave as  $N_c$  increases from 3 within a unitarized chiral approach. We obtain the result that vector mesons such as  $\rho$  and  $K^*$  survive as narrow width resonances, but all of the scalar meson nonet below 1 GeV fade out as  $N_c$  exceeds a rather small number about 6.

## 1 Introduction

It has been shown by Peláez [1] that the complex poles corresponding to the  $\rho$  and  $K^*$  mesons move toward the real axis on the second Riemann sheet as a number of colors,  $N_c$ , becomes large, and that in contrast to the vector mesons those corresponding to the  $\sigma$  and  $\kappa$  states move away from the real axis, within the inverse amplitude method (IAM) using full  $O(p^4)$  amplitudes of the chiral perturbation theory (ChPT)[2, 3, 4, 5, 6].

Stimulated by this work, we have calculated how physical quantities such as phase shifts and cross sections behave on the real axis of the physical sheet as  $N_c$  increases from 3 to some finite values[7]. In this calculation we adopt an approximate version of the two-channel IAM developed by Oller-Oset-Peláez (OOP)[8, 9], which we call the OOP version. We have obtained the result that while the vector mesons become narrower resonances, the light scalar meson nonet including the  $f_0(980)$  and  $a_0(980)$  states fade out as  $N_c$  exceeds about 6. This result has been confirmed by Peláez in his new paper, excluding an exceptional case of the  $a_0(980)$  state[10].

The same issue have been discussed by Oller and Oset[11] using a different model, in which chiral  $O(p^2)$  amplitudes and possible preexisting tree resonance poles are introduced as ingredients of the model. Their criterion whether a meson is dynamical or not is that the partial wave amplitude of the model can reproduce the experimental data without a nearby preexisting pole. The preexisting poles are assumed to survive in the large  $N_c$  limit. They have concluded that while the  $\rho$  and  $K^*$  mesons need each preexisting pole, light scalar mesons, possibly except for the  $f_0(980)$  state, do not necessarily need such poles. We note, however, that if energies to be fitted by the model are restricted to 1.2 GeV, the preexisting pole is not needed for the  $f_0$  state[12].

Thus, the above observations are consistent with the common understanding that the members of the vector meson nonet including  $\rho$  and  $K^*$  are typical of  $q\bar{q}$  mesons in large  $N_c$  QCD[13, 14]. On the other hand the behavior of scalar mesons is at variance with the nature of the  $q\bar{q}$  mesons.

In this paper we study again the behavior of two-meson scattering amplitudes when  $N_c$  increases from 3 under some different conditions from previous work, within the two-channel OOP version with  $O(p^4)$  amplitudes given in Ref. [5]. The  $O(p^4)$  amplitudes depend

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on values of the low energy constants (LEC) of ChPT, denoted as  $L_n$ , which are to be determined phenomenologically so as to reproduce experimental data. It would be difficult, therefore, to discriminate the nature of resonances by studying the  $N_c = 3$  world alone. In order to know the nature of the low mass mesons, it will be very useful to study how light vector and scalar meson states behave and how complex poles of  $f_0(980)$  and  $a_0(980)$  move when we increase  $N_c$  from 3 to some values. Although we do not go far away from the real  $N_c = 3$  world with  $SU(3) \times SU(3)$  chiral symmetry, we observe that the  $\rho$  and  $K^*$  meson survive as narrow width resonances, but the light scalar nonet fade out as  $N_c$  exceeds a rather small number about 6. This is the same result as in the previous work[7].

In the next section the explicit  $N_c$  dependence of the OOP amplitudes is given, the vector channel and scalar channel are discussed in section 2 and 3, and the conclusions and discussion is given in the last section.

## 2 $N_c$ dependence of the OOP amplitudes

In order to carry out the study we have to find the explicit  $N_c$  dependence of the scattering amplitudes. The amplitudes in ChPT have an explicit  $N_c$  dependence through the pion decay constant and the LECs. Since the pion decay constant  $f_\pi$  is of  $O(N_c^{1/2})$  and the LECs,  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_5$  and  $L_8$  are to be of  $O(N_c)$ , but  $2L_1 - L_2$ ,  $L_4$ ,  $L_6$  and  $L_7$  are of  $O(1)$ , [15, 16, 17] we put

$$L_n(N_c) = \widehat{L}_n \cdot \frac{N_c}{3} + \Delta L_n, \quad (1)$$

$$f_\pi^2(N_c) = \widehat{f}_\pi^2 \cdot \frac{N_c}{3}, \quad (2)$$

where  $\widehat{L}_n$  satisfy the relations,  $2\widehat{L}_1 - \widehat{L}_2 = \widehat{L}_4 = \widehat{L}_6 = \widehat{L}_7 = 0$ , and  $\Delta L_n$  are of  $O(1)$ . Thus, we have

$$\frac{L_n}{f_\pi^2} = \frac{\widehat{L}_n}{\widehat{f}_\pi^2} + \frac{\Delta L_n}{\widehat{f}_\pi^2} \cdot \frac{3}{N_c} \quad (3)$$

with  $\widehat{f}_\pi = 93$  MeV. We also assume that the meson decay constants are the same and equal to the pion decay constant  $\widehat{f}_\pi$  as in Ref. [5].

The ingredients of the IAM consist of amplitudes of chiral order  $O(p^2)$  and  $O(p^4)$  of ChPT. An  $O(p^2)$  amplitude, denoted by  $T^{(2)}(s, t, u)$ , has a form of a linear function of  $s$ ,  $t$ ,  $u$  divided by  $f_\pi^2$ , and it is of  $O(N_c^{-1})$ . A polynomial term of the latter amplitudes, denoted by  $T_{\text{poly}}^{(4)}(s, t, u)$ , is written as a sum of polynomial functions with the LECs as follows:

$$T_{\text{poly}}^{(4)}(s, t, u) = \sum_{n=1,8} \frac{1}{f_\pi^2} \left( \frac{L_n}{f_\pi^2} \right) P_n(s, t, u), \quad (4)$$

where  $P_n$  are quadratic functions of  $s$ ,  $t$ ,  $u$  and meson mass squared. The polynomial term  $T_{\text{poly}}^{(4)}$  is of  $O(N_c^{-1})$ , because  $L_n/f_\pi^2$  scales as  $O(N_c^0)$  as seen in Eq.(3). An  $s$ -channel loop term given by  $t^{(2)}(s)J(s)t^{(2)}(s)$  is of  $O(N_c^{-2})$ , where  $J(s)$  is the one-loop function regularized as the  $\overline{MS} - 1$  scheme at the renormalization scale  $\mu$  [15], where  $t^{(2)}$  is a partial wave amplitude derived from  $T^{(2)}(s, t, u)$ . Similarly  $t$ - and  $u$ -channel loop terms and tadpole terms are of  $O(N_c^{-2})$ , which are ignored in the OOP version. Thus, the OOP version is expected to be more valid as  $N_c$  becomes larger. The  $s$ -channel loop terms are indispensable to realize unitarity, although they are of  $O(N_c^{-2})$ . This difference of the  $N_c$  dependence produces the different behavior of the amplitudes when  $N_c$  becomes large. Our set of the LECs used in this work are determined at the renormalization scale  $\mu = 900$  MeV so as to reproduce experimental phase shifts qualitatively up to about 1.2 GeV. We note that the IAM and OOP amplitudes contain the LECs non-linearly and the fitting region is extended to higher energies. Our sets of  $\widehat{L}_n$  and  $\Delta L_n$  are tabulated in Table I with the set of the large  $N_c$

	$L_1$	$L_2$	$L_3$	$L_5$	$L_7$	$L_8$
Large $N_c$	0.81	1.62	-4.24	1.21	0	0.60
Our $\widehat{L}_n$	0.70	1.40	-3.20	1.50	0	0.71
Our $\Delta L_n$	0	-0.10	0	0	-0.25	0

Table 1:  $L_n \times 10^3$

model,[18, 19] which correspond to  $\widehat{L}_n$ . The change of the renormalization scale affects values of  $\Delta L_n$ , but we do not consider the scale change explicitly because the  $\Delta L_n$  terms fade out as  $N_c$  increases.

We emphasize that since the Large  $N_c$  set can reproduce the low energy scattering behavior rather well even at  $N_c = 3$ , except for unessential points, the results are almost the same if we take Large  $N_c$  set instead of Our  $\widehat{L}_n$  and put  $\Delta L_n$  to the difference between our  $L_n$  and the Large  $N_c$  set.

### 3 Vector channels

At first, we discuss the behavior of vector mesons in the single channel calculation. The mass of a vector resonance is controlled by the combination of LEC,  $2L_1 - L_2 + L_3$ , [2] which is present in the term  $\text{Re}[t^{(2)} - t_{\text{poly}}^{(4)}]$  of  $O(N_c^{-1})$ , and the loop contribution to the real part is of  $O(N_c^{-2})$ , so that the mass stays at an almost constant value. This is the IAM expression substituting for the preexisting pole. On the other hand the imaginary part of the loop term contributes to the width. The octet component of the isoscalar vector meson

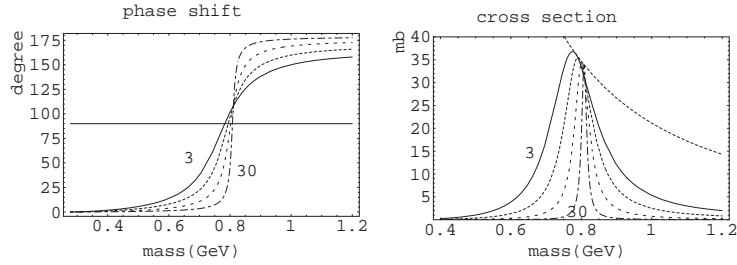


Figure 1:  $N_c$  dependence of phase shift (left) and cross section (right) of the  $\rho$  channel. Lines correspond to  $N_c = 3, 5, 10$  and  $30$  from the top to the bottom.

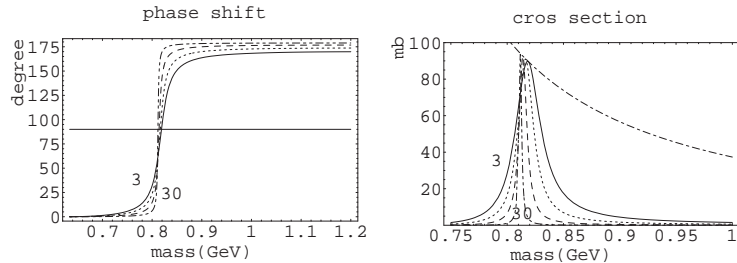


Figure 2:  $N_c$  dependence of phase shift (left) and cross section (right) of the  $K^*$  channel. Lines correspond to  $N_c = 3, 5, 10$  and  $30$  from the top to the bottom.

also has a constant mass and a residue of  $O(N_c^{-1})$  below the  $K\bar{K}$  threshold. Thus, the masses of the vector mesons are of  $O(1)$ , while the widths decrease as  $O(N_c^{-1})$ .

If we extend the calculation to the multi-channel IAM, the masses are almost unchanged, because the same combination of the LECs dominantly determine the masses. The  $N_c$  dependence of the phase shifts and that of the cross sections of  $\rho$  and  $K^*$  are shown in Figs. 1 and 2. The same behavior is observed in the case of the Large  $N_c$  set, though the obtained masses are too small because of too large value of  $|L_3|$ .<sup>1</sup>

Thus, we can conclude that the vector mesons described by the OOP version have the nature consistent with the  $q\bar{q}$  mesons.

## 4 Scalar channels

### 4.1 $(I, J) = (0, 0)$

This channel contains the controversial  $\sigma(600)$  and the  $f_0(980)$  states. Using the two-channel IAM consisting of the  $\pi\pi$  and  $K\bar{K}$  channels, we can reproduce experimental data fairly well below 1.2 GeV. The  $N_c$  dependence of the phase shift and the cross section are shown in Fig. 3, where  $N_c$  increases from 3 to 15. In contrast to the vector channel we observe that the phase shift becomes flat and the cross section fades out as  $N_c$  becomes large; the sharp rise of the phase shift near the  $K\bar{K}$  threshold and the large bump of the cross section near

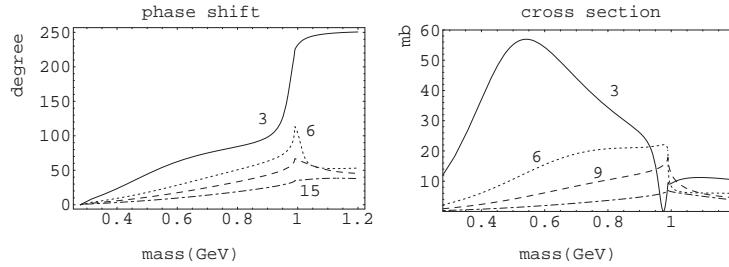


Figure 3:  $N_c$  dependence of the phase shift (left) and the cross section (right) of the (0,0) channel. Solid, dotted, dot-dot-dashed and dashed lines are for  $N_c=3, 6, 9$  and 15 respectively.

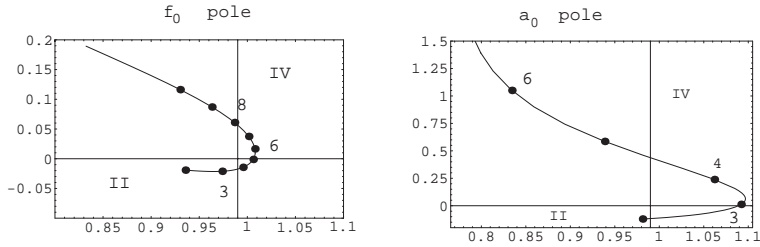


Figure 4:  $N_c$  dependence of the  $f_0(980)$  pole (left) and  $a_0(980)$  pole (right). Both of the poles wind around the branch point at  $K\bar{K}$  threshold to go upward on the IV sheet.

<sup>1</sup>The constraint  $2L_1 = L_2$  induces a large cancellation in  $\det[t^{(2)} - t^{(4)} - \text{loop}]$  even at  $N_c = 3$ , so that the resultant amplitude gets a large uncertainty. But the narrowing width with increasing values of  $N_c$  remains valid.

500 MeV seen at  $N_c = 3$  to 5 disappears at  $N_c = 6$ , and then the phase shift and the cross section become almost flat and fade out. Similar drastic change in the  $N_c$  dependence has also been observed in Ref. [20], though it is in different context.

The  $f_0(980)$  pole exists at  $(975 - 22i)$  MeV at  $N_c = 3$ . Where does the pole go as  $N_c$  increases? We approximately calculate the pole position by expanding the amplitude in powers of  $k_2$  up to the first order, where  $k_2$  is the momentum of the  $K\bar{K}$  channel. We observe that the pole moves into the upper half plane of the IV sheet from the lower half plane of the II sheet, winding around the branch point at  $K\bar{K}$  threshold, and goes away from the real axis as shown in the left side of Fig. 4. Pole positions at larger  $N_c$  cannot be reliable owing to the rough approximation, but this behavior would remain intact.

We briefly comment on effects of adding the  $\eta\eta$  channel to the OOP amplitude. If we include the  $\eta\eta$  channel, we find that both real and imaginary part of  $\det[t^{(2)} - t^{(4)} - \text{loopterms}]$  develop zeros at almost the same point near 770 MeV even at  $N_c = 3$  for a wide range of the LEC sets. Such unreasonable behavior is also seen in the isospinor channel with the  $\pi K$  and  $\eta K$  channels as will be noted. These zeros do not violate unitarity, but give very unreasonable behavior to the amplitude, that is too narrow resonant behavior with almost zero width. It should be noted, however, that the behavior of the amplitude, excluding a narrow strip including the zeros, is almost the same with the behavior of the two-channel model except for a shallow dip of the inelasticity at the  $\eta\eta$  threshold[12]. The unpleasant behavior is not reported in the calculations with full  $T^{(4)}$ [5, 10]. If we eliminate the unreasonable behavior, the fading-out tendency as increasing  $N_c$  remain valid, though the fading-out occurs at larger  $N_c$  owing to the additional  $\eta$  loop contributions.

#### 4.2 $(I, J) = (1, 0)$

This channel contains the  $a_0(980)$  state and appears as a cusp-like sharp peak as seen in Fig. 5.

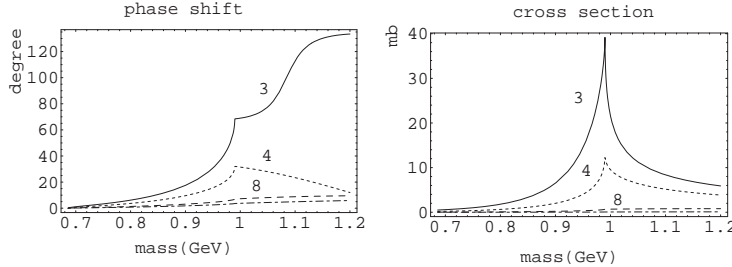


Figure 5:  $N_c$  dependence of the phase shift (left) and the cross section (right) of the (1,0) channel.  $N_c = 3, 4, 8$  and  $12$  from the top to bottom.

The rising phase shift after the cusp bends down and becomes to a flat curve, and the cross section having a sharp peak fades out as  $N_c$  increases. The pole appears at  $(1091 - 17i)$  MeV on the II sheet at  $N_c = 3$ , and it moves from the II sheet to the IV sheet, and leaves rapidly the real axis as shown in the left side of Fig. 4 as  $N_c$  increases. We note that the real parts of the poles of the  $f_0(980)$  and  $a_0(980)$  states are not necessarily degenerate with each other, though both of the peaks of the mass distribution at  $N_c = 3$  appear near the  $K\bar{K}$  threshold owing to the cusp behavior of the  $a_0(980)$  state.

#### 4.3 $(I, J) = (1/2, 0)$

The fading-out behavior of the phase shift and cross section with increasing  $N_c$  is the same as that in the channels discussed above. As stated before there appears an artifact zero near

750 MeV originated from the  $\eta K \rightarrow \eta K$  component in the OOP version used in this work. So if we eliminate the zero by an interpolation method, we observe that the results by the two channel model are almost the same as those by the single channel calculation by virtue of the weak coupling between the  $\pi K$  and  $\eta K$  channels. The calculations making use of the full  $T^{(4)}$  do not give such an unwanted zero [5].

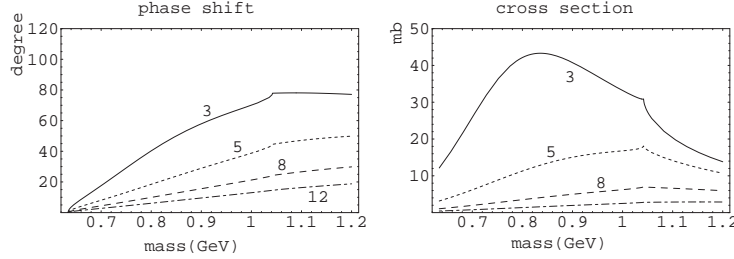


Figure 6:  $N_c$  dependence of phase shift (left) and cross section (right) of the  $(1/2,0)$  channel.  $N_c = 3, 5, 8$  and  $12$  from the above to bottom.

## 5 Concluding remarks

We have calculated the  $N_c$  dependence of the vector and scalar channels stating from  $N_c = 3$  to finite values, 30 for the vector channel and 12 or 15 for the scalar channel within the approximate IAM under the  $N_c$  dependence of  $L_n/f_\pi^2$  given by Eq.(3) and  $f_\pi(N_c) = \sqrt{N_c/3} \times f_\pi(3)$ . And we have observed that the vector mesons survive as sharper resonances at almost the same position, the resonant structures of the scalar channels fade out at rather low values of  $N_c$  near 5 or 6. By extending the observation we are led to conclude that the vector meson nonet has the nature consistent with the  $q\bar{q}$  mesons in large  $N_c$  QCD, but the light scalar meson nonet cannot survive in large  $N_c$  and then cannot have the nature of the  $q\bar{q}$  mesons. This conclusion is the same as obtained by Peláez, excluding an exceptional case of the  $a_0(980)$  state,[10] and it is also consistent with the results obtained in Ref. [11, 12]. Our conclusion supports the arguments that the light scalar mesons are of the  $K\bar{K}$  molecule[21, 22, 23], and of  $q^2\bar{q}^2$  states[24, 25, 26].

If the  $f_0$  and  $a_0$  states are composed of  $(qs)(\bar{q}\bar{s})$  state, where  $q$  denotes  $u$  and/or  $d$  quark, the similarity between the  $f_0$  and  $a_0$  both in mass and generating mechanism is expected. However, the pole positions of the both states can be different from each other by about 100 MeV or more, and the generating mechanism of the  $a_0(980)$  state would be different from that of  $f_0$  state: The  $a_0$  state is generated by the strong channel coupling between  $\pi\eta$  and  $K\bar{K}$  channels, but not as a bound state resonance like as the  $f_0$  state, as seen in the exchange dynamics[21, 23] and in the chiral loop dynamics[27, 9]. There is also the argument that the  $a_0(980)$  and  $f_0(980)$  are not elementary particles within the hadronic dynamics[28]. They have observed that the field renormalization constants  $Z$  of both states are close to 0 using the propagators of existing models, and concluded that a simple  $q\bar{q}$  or four quark assignment for the  $a_0$  should be considered with caution and it is certainly questionable for the  $f_0$ . If the light scalar nonet are not  $q\bar{q}$  mesons, we cannot include them into mass spectra in the low meson dominance hypothesis, because the low mesons are supposed to participate in narrow resonance towers in large  $N_c$  limit of QCD[18, 19, 29, 30, 31].

Our conclusion strongly indicates that all of the light scalar nonet are dynamical effects originating from unitarity, chiral symmetry and strong channel couplings. If the mesons in the scalar nonet are dominantly composed of hadronic or four quark component, but include  $|q\bar{q}\rangle_P$  with a small fraction as in Ref. [32], we could find out the small  $|q\bar{q}\rangle_P$  component by increasing  $N_c$  in theoretical models, because the large hadronic or four quark component

fades out and the  $q\bar{q}$  component remains. At least, our calculation within the two-channel OOP approximation does not indicate that such an intriguing change will occur in larger  $N_c$  region.

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